REAP 1.11.5

Users guide

S. Antusch, J. Kersten, M. Lindner, M. Ratz and M.A. Schmidt

December 21, 2021

This is an users guide of the `REAP` add-on for Mathematica. We describe the functions which allow to calculate the evolution of the neutrino mass matrix in different models (SM, MSSM, 2HDM). There is a reference of the most important functions. Besides this function reference there is a description of the installation procedure, a short HowTo and a section about frequently asked questions.

Contents

1 Introduction

The `REAP` (**R**enormalization group **E**volution of **A**ngles and **P**hases) package is a Mathematica package to solve the renormalization group equations (RGE) of the quantities relevant for neutrino masses, for example the dimension 5 neutrino mass operator, the Yukawa matrices and the gauge couplings. So far, the β-functions for the standard model (SM), the minimal supersymmetric standard model (MSSM) and the two higgs doublet model with $\mathbb{Z}_2$ symmetry (2HDM) with and without right-handed neutrinos are implemented. Heavy degrees of freedom such as singlet neutrinos can be integrated out automatically at the correct mass thresholds which are determined by a fixed-point iteration. Thus the evolution is described by several effective theories. In addition all models are implemented with Dirac neutrinos. By means of the `MixingParameterTools` package, the calculated running of the neutrino mass matrix can be translated into the running of the mixing parameters and the mass eigenvalues.
If you would like to refer to REAP in a publication or talk, please cite the accompanying paper hep-ph/0501272.

2 Installation

2.1 Automatic Installation under UNIX/Linux

Execute REAP.installer and you are done.

```
sh REAP.installer
```

After the execution the Mathematica packages are copied to `~/.Mathematica/Applications/REAP` and the documentation and notebooks are in a subdirectory of the working directory, which is called REAP.
In addition, you have to install the package MixingParameterTools. There is also a script which installs both REAP and MPT at the same time: REAP_MPT.installer.

2.2 Semi-Automatic Installation under UNIX/Linux

Unpack the archive REAP.tar.gz.

```
tar -xvzf REAP.tar.gz
```

Then go to the directory REAPInstall and execute the script install.sh.

```
cd REAPInstall
sh ./install.sh
```

The script copies the Mathematica packages to `~/.Mathematica/Applications/REAP`. The documentation and some sample notebooks are placed in a new subdirectory of the working directory called REAP. Hence, the folder REAPInstall can be deleted now.
In addition, you have to install the package MixingParameterTools. REAP and MPT can be installed simultaneously by using the archive REAP_MPT.tar.gz. The procedure is completely analogous to the one described above.

2.3 Installation by Hand

In order to install the package(s) manually, unpack the archive REAP.tar.gz first. Under UNIX/Linux, type

```
tar -xvzf REAP.tar.gz
```

On Windows systems, a program like WinZip can be used. Then move the directory REAP from the folder REAPInstall to the directory where the Mathematica add-ons are located, e.g.

```
mv REAPInstall/REAP ~/.Mathematica/Applications/
```

under UNIX/Linux. Under Windows XP, the path to the add-on directory should be something like `Application Data\Mathematica\Applications`. The documentation and some sample notebooks can be found in REAPInstall/Doc/REAP/.
In addition, you have to install the package MixingParameterTools. To install both REAP and MPT at the same time, you can use the archive REAP_MPT.tar.gz. The procedure is the same as above (except that the installation directory is called REAP_MPTInstall now), supplemented by an analogous step for moving the MPT directory, e.g.

```
mv REAP_MPTInstall/MixingParameterTools ~/.Mathematica/Applications/
```

3 First Steps

The following simple example demonstrates how to calculate the RG evolution of the neutrino mass matrix in the MSSM extended by three heavy singlet neutrinos.

(1) The package corresponding to the model at the highest energy has to be loaded. All other packages needed in the course of the calculation are loaded automatically.

```
Needs["REAP`RGEMSSM`"]
```

Note that ‘ is the backquote, which is used in opening quotation marks, for example.

(2) Next, we specify that we would like to use the MSSM with singlet neutrinos:

```
RGEAdd["MSSM"]
```

(3) Now we have to provide the initial values. Here we use the default values of the package (see Sec. 5 for details) and a simple diagonal pattern for the neutrino Yukawa matrix.

```
RGESetInitial[2*10^16,RGEY\[Nu]->{{1,0,0},{0,0.5,0},{0,0,0.1}}]
```

(4) `RGESolve[low,high]` solves the RGEs between the energy scales low and high. The heavy singlets are integrated out automatically at their mass thresholds.

```
RGESolve[100,2*10^16]
```

(5) Using `RGEGetSolution[scale,`*`quantity`*`]` we can query the value of the quantity given in the second argument at the energy given in the first one. For example, this returns the mass matrix of the light neutrinos at 100 GeV:

```
MatrixForm[RGEGetSolution[100,RGEM\[Nu]]]
```

(6) To find the leptonic mass parameters, we use the function `MNSParameters[`m_ν`,`Y_e`]` (which also needs the Yukawa matrix of the charged leptons). The results are given in the order $\{\{\theta_{12},\theta_{13},\theta_{23},\delta,\delta_e,\delta_\mu,\delta_\tau,\varphi_1,\varphi_2\},\{m_1,m_2,m_3\},\{y_e,y_\mu,y_\tau\}\}$.

```
MNSParameters[RGEGetSolution[100,RGEM\[Nu]],RGEGetSolution[100,RGEYe]]
```

(7) Finally, we can plot the running of the mixing angles:

```
Needs["Graphics`Graphics`"]
mNu[x_]:=RGEGetSolution[x,RGEM\[Nu]]
Ye[x_]:=RGEGetSolution[x,RGEYe]
\[Theta]12[x_]:=MNSParameters[mNu[x],Ye[x]][[1,1]]
\[Theta]13[x_]:=MNSParameters[mNu[x],Ye[x]][[1,2]]
\[Theta]23[x_]:=MNSParameters[mNu[x],Ye[x]][[1,3]]
LogLinearPlot[{\[Theta]12[x],\[Theta]13[x],\[Theta]23[x]},{x,100,2*10^16}]
```

To produce nicer plots, the notebook RGEPlots.nb, which is included in the package, can be used.

In a second run, let us try some more modifications of the defaults. For example, model parameters can be changed by including a command after step (2):

```
RGESetOptions["MSSM",RGEtan\[Beta]->20]
```

Furthermore, we set the SUSY breaking scale to 200 GeV and use the SM as an effective theory below this scale.

```
RGEAdd["SM",RGECutoff->200]
```

The initial values of the neutrino mass parameters can be changed by adding replacement rules in step (3). For instance, to set the GUT-scale value of θ_{13} to 6° and the Majorana phases to 50° and 120°:

```
RGESetInitial[2*10^16,
  RGEY\[Nu]->{{1,0,0},{0,0.5,0},{0,0,0.1}},RGE\[Theta]13->6 Degree,
  RGE\[CurlyPhi]1->50 Degree,RGE\[CurlyPhi]2->120 Degree]
```

The results of the RG evolution with these parameters are now obtained by repeating the above steps (4)–(7).

4 Reference

4.1 Implementation details

`REAP` is divided in three parts. The main part is `RGESolver` which provides a standard interface between the different models and the user. Thus the user does not have to know anything about the implementation details of the different models besides the parameters of the models. The second part are the different models, like `RGESM`, `RGEMSSM`, . . . which contain the model specific parts of the package. The third part is formed by some utility packages (`RGEUtilities`, `RGEParameters`, `RGEInitial`, `RGEFusaokaYukawa`, `RGESymbol`, `RGETakagi`) which provide several useful functions to the different models. In principle, a user only needs a limited set of functions of `RGESolver`.

4.1.1 RGEAdd

`RGEAdd[model,`*`options`*`]` specifies that `model` should be used as an effective theory (EFT) up to a cutoff energy given in the *`options`*. If no cutoff is given, ∞ is used. *`options`* can also be used to specify various parameters such as $\tan\beta$. See Sec. 5 for a complete list of the models and options available.

```
RGEAdd["MSSM",RGEtan\[Beta]->50]
RGEAdd["SM",RGECutoff->10^3]
```

In this case, the MSSM with $\tan\beta = 50$ is used at high energies. Below 10^3 GeV (the SUSY breaking scale in this example), the SM is used as an EFT.

4.1.2 RGEGetOptions

`RGEGetOptions[model]` returns the options set by `RGEAdd` or `RGESetOptions` for the EFT `model`. Wildcards can be used in `model`.

```
RGEGetOptions["SM*"]
```

This returns the options which are currently set for all EFTs whose names start with "SM".

4.1.3 RGEGetParameters

`RGEGetParameters[model]` returns the quantities that run in the `model`.

4.1.4 RGEGetSolution

`RGEGetSolution[scale,`*`parameter`*`]` returns the solution of the RGEs at the energy `scale`. The *`parameter`* (optional) specifies the quantity of interest (cf. Sec. 5 for the lists for each model). If no *`parameter`* is given, the values of all running quantities are returned.

```
RGEGetSolution[100,RGEM\[Nu]]
```

returns the neutrino mass matrix at 100 GeV.

```
RGEGetSolution[100]
```

returns all parameters at 100 GeV.

4.1.5 RGEGetTransitions

`RGEGetTransitions[]` returns the transitions (thresholds) between the various EFTs in a list containing the energy scale, the model name and its options.

4.1.6 RGEReset

`RGEReset[]` removes all EFTs and resets all options which have been changed by `RGEAdd` or `RGESetOptions` to their default values.

4.1.7 RGESetInitial

`RGESetInitial[scale,initial conditions]` sets the initial values at the energy `scale`. They are entered as replacement rules and can also contain options (e.g. to select the neutrino mass hierarchy). See Sec. 5 for the names of the variables and options in the different models.

```
RGESetInitial[10^16,RGE\[Theta]13->4 Degree,RGEMlightest->0.1]
```

This sets the initial values at 10^{16} GeV. The mixing angle θ_{13} is set to $4°$, and the mass of the lightest neutrino to 0.1 eV. For the other parameters, the default values are used.

4.1.8 RGESetOptions

`RGESetOptions[model,options]` changes the options of the EFTs defined by `RGEAdd` with name matching `model` to `options`. Metacharacters, like * and @, are allowed in the name.

```
RGESetOptions["MSSM",RGEtan\[Beta]->40]
```

This sets $\tan\beta$ of the "MSSM" to 40. The EFT must have been added earlier by `RGEAdd["MSSM"]`. The other options are unchanged.

4.1.9 RGESolve

`RGESolve[low,high,options]` solves the RGEs between the energies `low` and `high`.

```
RGESolve[100,10^15]
```

This solves the RGEs between 100 GeV and 10^{15} GeV.

5 Models

5.1 Standard Model (SM)

5.1.1 REAP‘RGESM‘

This package contains the Standard Model extended by an arbitrary number of right-handed neutrinos (SM) to 1 loop order. It is possible to automatically find transitions where heavy neutrinos are integrated out. However, quarks are not integrated out.

Options used by `RGESetInitial`:
If the default values of all parameters are used, the resulting parameters will be compatible to the experimental data at the Z boson mass. The number of right-handed neutrinos is given by the initial conditions. There is no need to specify the number of neutrinos somewhere else.

- RGEMνr is the mass matrix of the right-handed neutrinos. If this parameter is specified, it also determines the light neutrino mass matrix via the see-saw formula (together with RGEYν). Thus, RGEMassHierarchy, RGEMlightest, RGEΔm2atm, RGEΔm2sol, RGEφ1, RGEφ2, RGEδ, RGEδe, RGE$\delta\mu$, RGE$\delta\tau$, RGEθ12, RGEθ13, and RGEθ23 do not have any effect in this case.
- RGEMassHierarchy is the hierarchy of the neutrino masses; ”r” or ”n” means normal hierarchy, ”i” means inverted hierarchy (default: ”r”).
- RGEMlightest is the mass of the lightest neutrino in eV (default: $\mathcal{O}(0.01)$ eV).
- RGEYν is the neutrino Yukawa matrix in “RL convention”. This option overrides the built-in Yukawa matrix, i.e. RGEYν33 and RGEYνRatio do not have any effect.
- RGEYν33 is the (3,3) entry in the neutrino Yukawa matrix at the GUT scale.
- RGEYνRatio determines the relative value of the neutrino Yukawa couplings.
- RGEYd is the Yukawa matrix of the down-type quarks. If this parameter is given, RGEyd, RGEys, RGEyb, RGEqφ1, RGEqφ2, RGEqδ, RGEqδe, RGEq$\delta\mu$, RGEq$\delta\tau$, RGEqθ12, RGEqθ13, and RGEqθ23 are ignored.
- RGEYe is the charged lepton Yukawa matrix. If this parameter is given, RGEye, RGEyμ and RGEyτ are ignored.
- RGEYu is the Yukawa matrix of the up-type quarks. If this parameter is given, RGEyu, RGEyc and RGEyt are ignored; it is recommended not to use RGEqφ1, RGEqφ2, RGEqδ, RGEqδe, RGEq$\delta\mu$, RGEq$\delta\tau$, RGEqθ12, RGEqθ13, and RGEqθ23 in this case, since they are not necessarily equal to the CKM mixing parameters.
- RGEΔm2atm is the atmospheric mass squared difference (default: $\mathcal{O}(10^{-3})\,\mathrm{eV}^2$).
- RGEΔm2sol is the solar mass squared difference (default: $\mathcal{O}(10^{-4})\,\mathrm{eV}^2$).
- RGEφ1 and RGEφ2 are the Majorana CP phases φ_1 and φ_2 in radians (default: 0).
- RGEδ is the Dirac CP phase δ in radians (default: 0).

- RGEδe, RGE$\delta\mu$ and RGE$\delta\tau$ are the unphysical phases δ_e, δ_μ and δ_τ (default: 0).
- RGEκ is the coupling of the dimension 5 neutrino mass operator.
- RGEλ is the quartic Higgs self-coupling (default: 0.5). We use the convention that the corresponding term in the Lagrangian is $-\frac{\lambda}{4}(\phi^\dagger\phi)^2$.
- RGEθ12, RGEθ13 and RGEθ23 are the angles θ_{12}, θ_{13} and θ_{23} of the MNS matrix in radians. (default: $\theta_{13} = 0$ and $\theta_{23} = \frac{\pi}{4}$). The default of θ_{12} depends on the model. It is chosen in such a way, that the parameters are compatible with the experimental data.
- RGEg RGEg is the coupling constants of SU(5)
- RGEg1, RGEg2 and RGEg3 are the coupling constants of $\mathrm{U(1)_Y}$, $\mathrm{SU(2)_L}$ and $\mathrm{SU(3)_C}$, respectively. GUT charge normalization is used for g_1.
- RGEm RGEm is the Higgs mass
- RGEqφ1 and RGEqφ2 are the unphysical phases φ_1 and φ_2 of the CKM matrix which correspond to the Majorana phases in the MNS matrix (default: 0).
- RGEqδ is the Dirac CP phase δ of the CKM matrix.
- RGEqδe, RGEq$\delta\mu$ and RGE$\delta\tau$ are the unphysical phases δ_e, δ_μ and δ_τ of the CKM matrix (default: 0).
- RGEqθ12, RGEqθ13 and RGEqθ23 are the angles of the CKM matrix.
- RGEyd, RGEys and RGEyb are the Yukawa coupling of the down-type quarks d, s and b.
- RGEye, RGEyμ and RGEyτ are the Yukawa couplings of the charged leptons e, μ and τ.
- RGEyu, RGEyc and RGEyt are the Yukawa couplings of the up-type quarks u, c and t.

Parameters accepted by `RGEGetSolution`:

- RGECoupling is used to get the coupling constants.
- RGEGWCondition returns the Gildener Weinberg condition.
- RGEGWConditions returns all Gildener Weinberg conditions.
- RGEMν is used to get the mass matrix of the left-handed neutrinos.
- RGEMνr is the mass matrix of the right-handed neutrinos.
- RGEMd is used to get the mass matrix of the down-type quarks.
- RGEMe is used to get the mass matrix of the charged leptons.
- RGEMu is used to get the mass matrix of the up-type quarks.
- RGERawYΔ is used to get the Yukawa coupling matrix of the coupling to the Higgs triplet.
- RGEAll returns all parameters of the model.
- RGEVEVratio returns the squared ratio of v_R over the EW symmetry breaking scale.
- RGEVEVratios returns the squared ratio of v_R over the EW symmetry breaking scale.
- RGEYν is used to get the Yukawa coupling matrix of the neutrinos.
- RGEYd is used to get the Yukawa coupling matrix of the down-type quarks.

- RGEYe is used to get the Yukawa coupling matrix of the charged leptons.
- RGEYu is used to get the Yukawa coupling matrix of the up-type quarks.
- RGEα is used to get the fine structure constants.
- RGEλ is used to get the quartic Higgs self coupling.

5.1.2 `REAP‘RGESM0N‘`

This package contains the Standard Model without any right-handed neutrinos (SM0N) to 1 loop order.

It has the same parameters and options as `RGESM`, with the following exceptions: The only missing options are RGEIntegratedOut, RGESearchTransition, RGEThresholdFactor, RGEPrecision and RGEMaxNumberIterations, which are used to control the process of integrating out. Besides, RGEMνr and RGEYν are no parameters of `RGESetInitial`, and RGEϵMax, RGEϵ, RGEM1Tilde, RGERawMνr and RGERawYν are not accepted as parameters by `RGEGetSolution`. `RGESetInitial` has an additional option: RGESuggestion can be used to choose between different sets of default values, “GUT” (default) and “MZ”. They refer to typical parameter values at the GUT scale or at the Z mass, respectively.

5.1.3 `REAP‘RGESMDirac‘`

This package contains the Standard Model with Dirac Neutrinos to 1 loop order.

It has the same parameters and options as `RGESM`, with the following exceptions: The only missing options are RGEIntegratedOut, RGESearchTransition, RGEThresholdFactor, RGEPrecision and RGEMaxNumberIterations, which are used to control the process of integrating out. In addition RGEκ and RGEMνr are no parameters of `RGESetInitial` and RGEMixingParameters, RGEϵMax, RGEϵ, RGEM1Tilde, RGERawMνr, RGERawYν and RGEκ are not accepted as parameters by `RGEGetSolution`. `RGESetInitial` has an additional option: RGESuggestion can be used to choose between different sets of default values, “GUT” (default) and “MZ”. They refer to typical parameter values at the GUT scale or at the Z mass, respectively.

5.2 Minimal Supersymmetric Standard Model (MSSM)

5.2.1 `REAP‘RGEMSSM‘`

This package contains the Minimal Supersymmetric Standard Model extended by an arbitrary number of right-handed neutrinos (MSSM) to 1 and 2 loop order.
It is possible to automatically find transitions where heavy neutrinos are integrated out. But neither quarks are integrated out nor MSSM thresholds are considered.
Options:

- RGEΓd parameterizes the finite supersymmetric threshold corrections

  $$Y_d^{\mathrm{SM}} = Y_d^{\mathrm{MSSM}}(1 + \mathrm{RGE}\Gamma\mathrm{d}) * \cos(\beta) \tag{5.1}$$

  in the basis, in which Y_u is diagonal and the left-handed mixing is entirely contained in Y_d. It is related to the notation in [1]

  $$\mathrm{RGE}\Gamma\mathrm{d} \equiv \epsilon(V_{CKM}\Gamma_D^\dagger V_{CKM}^\dagger + \Gamma_U^\dagger) \tag{5.2}$$

  with $\epsilon = \tan\beta/(16\pi^2)$ and $\Gamma_{U,D}$ defines as in Eq. (1) of Ref. [1].

- RGEΓe parameterizes the finite supersymmetric threshold corrections

$$Y_e^{\mathrm{SM}} = Y_e^{\mathrm{MSSM}}(1 + \mathrm{RGE\Gamma e}) * \cos\beta \tag{5.3}$$

  in the basis, in which the Weinberg operator κ is diagonal and the left-handed mixing is entirely contained in Y_e. It is defined in a similar way to RGEΓd.

- RGEtanβ is the value of $\tan\beta = \frac{v_u}{v_d}$, the ratio of the 2 Higgs vevs (default: 50).

Options used by `RGESetInitial`:
If the default values of all parameters are used, the resulting parameters will be compatible to the experimental data at the Z boson mass. The number of right-handed neutrinos is given by the initial conditions. There is no need to specify the number of neutrinos somewhere else.

- RGEMνr is the mass matrix of the right-handed neutrinos. If this parameter is specified, it also determines the light neutrino mass matrix via the see-saw formula (together with RGEYν). Thus, RGEMassHierarchy, RGEMlightest, RGEΔm2atm, RGEΔm2sol, RGEφ1, RGEφ2, RGEδ, RGEδe, RGE$\delta\mu$, RGE$\delta\tau$, RGEθ12, RGEθ13, and RGEθ23 do not have any effect in this case.

- RGEMassHierarchy is the hierarchy of the neutrino masses; "r" or "n" means normal hierarchy, "i" means inverted hierarchy (default: "r").

- RGEMlightest is the mass of the lightest neutrino in eV (default: $\mathcal{O}(0.01)\,\mathrm{eV}$).

- RGEYν is the neutrino Yukawa matrix in "RL convention". This option overrides the built-in Yukawa matrix, i.e. RGEYν33 and RGEYνRatio do not have any effect.

- RGEYν33 is the (3,3) entry in the neutrino Yukawa matrix at the GUT scale.

- RGEYνRatio determines the relative value of the neutrino Yukawa couplings.

- RGEYd is the Yukawa matrix of the down-type quarks. If this parameter is given, RGEyd, RGEys, RGEyb, RGEqφ1, RGEqφ2, RGEqδ, RGEqδe, RGEq$\delta\mu$, RGEq$\delta\tau$, RGEqθ12, RGEqθ13, and RGEqθ23 are ignored.

- RGEYe is the charged lepton Yukawa matrix. If this parameter is given, RGEye, RGEyμ and RGEyτ are ignored.

- RGEYu is the Yukawa matrix of the up-type quarks. If this parameter is given, RGEyu, RGEyc and RGEyt are ignored; it is recommended not to use RGEqφ1, RGEqφ2, RGEqδ, RGEqδe, RGEq$\delta\mu$, RGEq$\delta\tau$, RGEqθ12, RGEqθ13, and RGEqθ23 in this case, since they are not necessarily equal to the CKM mixing parameters.

- RGEΔm2atm is the atmospheric mass squared difference (default: $\mathcal{O}(10^{-3})\,\mathrm{eV}^2$).

- RGEΔm2sol is the solar mass squared difference (default: $\mathcal{O}(10^{-4})\,\mathrm{eV}^2$).

- RGEφ1 and RGEφ2 are the Majorana CP phases φ_1 and φ_2 in radians (default: 0).

- RGEδ is the Dirac CP phase δ in radians (default: 0).

- RGEδe, RGE$\delta\mu$ and RGE$\delta\tau$ are the unphysical phases δ_e, δ_μ and δ_τ (default: 0).

- RGEκ is the coupling of the dimension 5 neutrino mass operator.

- RGEθ12, RGEθ13 and RGEθ23 are the angles θ_{12}, θ_{13} and θ_{23} of the MNS matrix in radians. (default: $\theta_{13} = 0$ and $\theta_{23} = \frac{\pi}{4}$). The default of θ_{12} depends on the model. It is chosen in such a way, that the parameters are compatible with the experimental data.

- RGEg RGEg is the coupling constants of SU(5)

- RGEg1, RGEg2 and RGEg3 are the coupling constants of $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively. GUT charge normalization is used for g_1.
- RGEm RGEm is the Higgs mass
- RGEqφ1 and RGEqφ2 are the unphysical phases φ_1 and φ_2 of the CKM matrix which correspond to the Majorana phases in the MNS matrix (default: 0).
- RGEqδ is the Dirac CP phase δ of the CKM matrix.
- RGEqδe, RGEq$\delta\mu$ and RGE$\delta\tau$ are the unphysical phases δ_e, δ_μ and δ_τ of the CKM matrix (default: 0).
- RGEqθ12, RGEqθ13 and RGEqθ23 are the angles of the CKM matrix.
- RGEyd, RGEys and RGEyb are the Yukawa coupling of the down-type quarks d, s and b.
- RGEye, RGEyμ and RGEyτ are the Yukawa couplings of the charged leptons e, μ and τ.
- RGEyu, RGEyc and RGEyt are the Yukawa couplings of the up-type quarks u, c and t.

Parameters accepted by `RGEGetSolution`:

- RGECoupling is used to get the coupling constants.
- RGEGWCondition returns the Gildener Weinberg condition.
- RGEGWConditions returns all Gildener Weinberg conditions.
- RGEMν is used to get the mass matrix of the left-handed neutrinos.
- RGEMνr is the mass matrix of the right-handed neutrinos.
- RGEMd is used to get the mass matrix of the down-type quarks.
- RGEMe is used to get the mass matrix of the charged leptons.
- RGEMu is used to get the mass matrix of the up-type quarks.
- RGERawYΔ is used to get the Yukawa coupling matrix of the coupling to the Higgs triplet.
- RGEAll returns all parameters of the model.
- RGEVEVratio returns the squared ratio of v_R over the EW symmetry breaking scale.
- RGEVEVratios returns the squared ratio of v_R over the EW symmetry breaking scale.
- RGEYν is used to get the Yukawa coupling matrix of the neutrinos.
- RGEYd is used to get the Yukawa coupling matrix of the down-type quarks.
- RGEYe is used to get the Yukawa coupling matrix of the charged leptons.
- RGEYu is used to get the Yukawa coupling matrix of the up-type quarks.
- RGEα is used to get the fine structure constants.
- RGEκ is used to get κ.

5.2.2 REAP‘RGEMSSMON‘

This package contains the Minimal Supersymmetric Standard Model (MSSM) without any right-handed neutrinos to 1 and 2 loop order.

It has the same parameter and options as `RGEMSSM`. The only missing options are RGEIntegratedOut, RGESearchTransition, RGEThresholdFactor, RGEPrecision and RGEMaxNumberIterations, which are used to control the process of integrating out. In addition RGEMνr and RGEYν are no parameters of `RGESetInitial` and RGEϵMax, RGEϵ, RGEM1Tilde, RGERawMνr and RGERawYν are not accepted as parameters by `RGEGetSolution`.

5.2.3 REAP‘RGEMSSMDirac‘

This package contains the MSSM with Dirac Neutrinos to 1 loop order and 2 loop order.

It has the same parameter and options as `RGEMSSM`. The only missing options are RGEIntegratedOut, RGESearchTransition, RGEThresholdFactor, RGEPrecision and RGEMaxNumberIterations, which are used to control the process of integrating out. In addition RGEMνr and RGEκ are no parameter of `RGESetInitial` and RGEMixingParameters, RGEϵMax, RGEϵ, RGEM1Tilde, RGERawMνr, RGERawYν and RGEκ are not accepted as parameters by `RGEGetSolution`.

5.3 Two Higgs Doublet Model (2HDM)

5.3.1 REAP‘RGE2HDM‘

This package contains the Two Higgs Doublet Model (2HDM) with a $\mathbb{Z}_2$ symmetry extended by an arbitrary number of right-handed neutrinos. The charged leptons always couple to the first Higgs. In addition there are right-handed neutrinos. The β-functions are to 1 loop order. The vevs of the Higgs fields are $v_1 = \langle\phi_1\rangle$ and $v_2 = \langle\phi_2\rangle$. They obey $v^2 = v_1^2 + v_2^2$, $v_1 = v\cos\beta$ and $v_2 = v\sin\beta$, where v is the v.e.v. of the SM Higgs and β ($\tan\beta = \frac{v_2}{v_1}$, $\beta \in \left(0, \frac{\pi}{2}\right)$) is used to parametrize the Higgs vevs.
Thus there are 2 dimension 5 operators which give mass to the light neutrinos.

$$\mathcal{L}_\kappa^{(ii)} = \frac{1}{4}\kappa_{gf}^{(ii)}\overline{l_{Lc}^C}^g \epsilon^{cd}\phi_d^{(i)} l_{Lb}^f \epsilon^{ba}\phi_a^{(i)} + \text{h.c.} \qquad (i = 1 \; or \; 2)$$

The Higgs potential is

$$\begin{aligned}\mathcal{L}_{2Higgs} &= -\frac{\lambda_1}{4}\left(\phi^{(1)\dagger}\phi^{(1)}\right)^2 - \frac{\lambda_2}{4}\left(\phi^{(2)\dagger}\phi^{(2)}\right)^2 \\ &\quad -\lambda_3\left(\phi^{(1)\dagger}\phi^{(1)}\right)\left(\phi^{(2)\dagger}\phi^{(2)}\right) - \lambda_4\left(\phi^{(1)\dagger}\phi^{(2)}\right)\left(\phi^{(2)\dagger}\phi^{(1)}\right) \\ &\quad -\left[\frac{\lambda_5}{4}\left(\phi^{(1)\dagger}\phi^{(2)}\right)^2 + \text{h.c.}\right]\end{aligned}$$

The charged leptons always couple to the first Higgs field and the coupling of the other fields to the Higgs fields is determined by RGEModelOptions.
It is possible to automatically find transitions where heavy neutrinos are integrated out. But no other particles are integrated out.
Options:

- RGEtanβ is the value of $\tan\beta = \frac{v_2}{v_1}$, the ratio of the 2 Higgs vevs (default: 50).
- RGEzν is a list defining the Higgs the neutrinos are coupling to. If the n^{th} component is one, the Higgs couples to the neutrinos. If it is 0, it won't couple (default: $\{0, 1\}$). The charged leptons always couple to the first Higgs.

- RGEzd is a list defining the Higgs the down-type quarks are coupling to. If the n^{th} component is one, the Higgs couples to the down-type quarks. If it is 0, it won't couple (default: $\{1, 0\}$).
- RGEzu is a list defining the Higgs the up-type quarks are coupling to. If the n^{th} component is one, the Higgs couples to the up-type quarks. If it is 0, it won't couple (default: $\{0, 1\}$).

Options used by `RGESetInitial`:
If the default values of all parameters are used, the resulting parameters will be compatible to the experimental data at the Z boson mass. The number of right-handed neutrinos is given by the initial conditions. There is no need to specify the number of neutrinos somewhere else.

- RGEMνr is the mass matrix of the right-handed neutrinos. If this parameter is specified, it also determines the light neutrino mass matrix via the see-saw formula (together with RGEYν). Thus, RGEMassHierarchy, RGEMlightest, RGEΔm2atm, RGEΔm2sol, RGEφ1, RGEφ2, RGEδ, RGEδe, RGE$\delta\mu$, RGE$\delta\tau$, RGEθ12, RGEθ13, and RGEθ23 do not have any effect in this case.
- RGEMassHierarchy is the hierarchy of the neutrino masses; "r" or "n" means normal hierarchy, "i" means inverted hierarchy (default: "r").
- RGEMlightest is the mass of the lightest neutrino in eV (default: $\mathcal{O}(0.01)$ eV).
- RGEYν is the neutrino Yukawa matrix in "RL convention". This option overrides the built-in Yukawa matrix, i.e. RGEYν33 and RGEYνRatio do not have any effect.
- RGEYν33 is the (3,3) entry in the neutrino Yukawa matrix at the GUT scale.
- RGEYνRatio determines the relative value of the neutrino Yukawa couplings.
- RGEYd is the Yukawa matrix of the down-type quarks. If this parameter is given, RGEyd, RGEys, RGEyb, RGEqφ1, RGEqφ2, RGEqδ, RGEqδe, RGEq$\delta\mu$, RGEq$\delta\tau$, RGEqθ12, RGEqθ13, and RGEqθ23 are ignored.
- RGEYe is the charged lepton Yukawa matrix. If this parameter is given, RGEye, RGEyμ and RGEyτ are ignored.
- RGEYu is the Yukawa matrix of the up-type quarks. If this parameter is given, RGEyu, RGEyc and RGEyt are ignored; it is recommended not to use RGEqφ1, RGEqφ2, RGEqδ, RGEqδe, RGEq$\delta\mu$, RGEq$\delta\tau$, RGEqθ12, RGEqθ13, and RGEqθ23 in this case, since they are not necessarily equal to the CKM mixing parameters.
- RGEΔm2atm is the atmospheric mass squared difference (default: $\mathcal{O}(10^{-3})\,\mathrm{eV}^2$).
- RGEΔm2sol is the solar mass squared difference (default: $\mathcal{O}(10^{-4})\,\mathrm{eV}^2$).
- RGEφ1 and RGEφ2 are the Majorana CP phases φ_1 and φ_2 in radians (default: 0).
- RGEδ is the Dirac CP phase δ in radians (default: 0).
- RGEδe, RGE$\delta\mu$ and RGE$\delta\tau$ are the unphysical phases δ_e, δ_μ and δ_τ (default: 0).
- RGEκ1 is the coupling of the dimension 5 operator associated with the first Higgs in the 2HDM.
- RGEκ2 is the coupling of the dimension 5 operator associated with the second Higgs in the 2HDM.
- RGEλ1, RGEλ2, RGEλ3, RGEλ4 and RGEλ5 are the parameters λ_1, λ_2, λ_3, λ_4 and λ_5 in the Higgs potential (default: $\lambda_1 = \lambda_2 = 0.75$, $\lambda_3 = \lambda_4 = 0.2$, $\lambda_5 = 0.25$).

- RGEθ12, RGEθ13 and RGEθ23 are the angles θ_{12}, θ_{13} and θ_{23} of the MNS matrix in radians. (default: $\theta_{13} = 0$ and $\theta_{23} = \frac{\pi}{4}$). The default of θ_{12} depends on the model. It is chosen in such a way, that the parameters are compatible with the experimental data.
- RGEg RGEg is the coupling constants of SU(5)
- RGEg1, RGEg2 and RGEg3 are the coupling constants of $\mathrm{U}(1)_\mathrm{Y}$, $\mathrm{SU}(2)_\mathrm{L}$ and $\mathrm{SU}(3)_\mathrm{C}$, respectively. GUT charge normalization is used for g_1.
- RGEm RGEm is the Higgs mass
- RGEqφ1 and RGEqφ2 are the unphysical phases φ_1 and φ_2 of the CKM matrix which correspond to the Majorana phases in the MNS matrix (default: 0).
- RGEqδ is the Dirac CP phase δ of the CKM matrix.
- RGEqδe, RGEq$\delta\mu$ and RGE$\delta\tau$ are the unphysical phases δ_e, δ_μ and δ_τ of the CKM matrix (default: 0).
- RGEqθ12, RGEqθ13 and RGEqθ23 are the angles of the CKM matrix.
- RGEyd, RGEys and RGEyb are the Yukawa coupling of the down-type quarks d, s and b.
- RGEye, RGEyμ and RGEyτ are the Yukawa couplings of the charged leptons e, μ and τ.
- RGEyu, RGEyc and RGEyt are the Yukawa couplings of the up-type quarks u, c and t.

Parameters accepted by `RGEGetSolution`:

- RGECoupling is used to get the coupling constants.
- RGEGWCondition returns the Gildener Weinberg condition.
- RGEGWConditions returns all Gildener Weinberg conditions.
- RGEMν is used to get the mass matrix of the left-handed neutrinos.
- RGEMνr is the mass matrix of the right-handed neutrinos.
- RGEMd is used to get the mass matrix of the down-type quarks.
- RGEMe is used to get the mass matrix of the charged leptons.
- RGEMu is used to get the mass matrix of the up-type quarks.
- RGERawYΔ is used to get the Yukawa coupling matrix of the coupling to the Higgs triplet.
- RGEAll returns all parameters of the model.
- RGEVEVratio returns the squared ratio of v_R over the EW symmetry breaking scale.
- RGEVEVratios returns the squared ratio of v_R over the EW symmetry breaking scale.
- RGEYν is used to get the Yukawa coupling matrix of the neutrinos.
- RGEYd is used to get the Yukawa coupling matrix of the down-type quarks.
- RGEYe is used to get the Yukawa coupling matrix of the charged leptons.
- RGEYu is used to get the Yukawa coupling matrix of the up-type quarks.
- RGEα is used to get the fine structure constants.

- RGEκ1 is the parameter of the dimension 5 operator associated with the first Higgs in the 2HDM.
- RGEκ2 is the parameter of the dimension 5 operator associated with the second Higgs in the 2HDM.
- RGEλ is used to get the Higgs couplings.

5.3.2 `REAP‘RGE2HDMON‘`

This package contains the Two Higgs Doublet Model (2HDM) with a $\mathbb{Z}_2$ symmetry without right-handed neutrinos.

It has the same parameters and options as `RGE2HDM`, with the following exceptions: The only missing options are RGEIntegratedOut, RGESearchTransition, RGEThresholdFactor, RGEPrecision and RGEMaxNumberIterations, which are used to control the process of integrating out. In addition RGEMνr and RGEYν are no parameters of `RGESetInitial` and RGEM1Tilde, RGERawMνr and RGERawYν are not accepted as parameters by `RGEGetSolution`. `RGESetInitial` has an additional option: RGESuggestion can be used to choose between different sets of default values, “GUT” (default) and “MZ”. They refer to typical parameter values at the GUT scale or at the Z mass, respectively.

5.3.3 `REAP‘RGE2HDMDirac‘`

This package contains the 2HDM with Dirac neutrinos to 1 loop order.

It has the same parameters and options as `RGE2HDM`, with the following exceptions: The only missing options are RGEIntegratedOut, RGESearchTransition, RGEThresholdFactor, RGEPrecision and RGEMaxNumberIterations, which are used to control the process of integrating out. In addition RGEMνr, RGEκ1 and RGEκ2 are no parameter of `RGESetInitial` and RGEMixingParameters, RGEM1Tilde, RGERawMνr, RGERawYν, RGEκ1 and RGEκ2 are not accepted as parameters by `RGEGetSolution`. `RGESetInitial` has an additional option: RGESuggestion can be used to choose between different sets of default values, “GUT” (default) and “MZ”. They refer to typical parameter values at the GUT scale or at the Z mass, respectively.

6 Frequently Asked Questions and their Answers (FAQ)

6.1 Physics Questions

6.1.1 How can I have more or less than 3 right-handed neutrinos?

The default initial values have 3 right-handed neutrinos but you can define a model with an arbitrary number of right-handed neutrinos by changing the initial values for the right-handed neutrino mass matrix and the Yukawa couplings of the neutrinos.

6.1.2 How are neutrino mixing parameters defined above the see-saw scale?

In order to define mass and mixing parameters as functions of the renormalization scale μ above the highest see-saw scale, we consider the effective light neutrino mass matrix

$$m_\nu(\mu) \;=\; -\frac{v^2}{2}\, Y_\nu^T(\mu)\, M^{-1}(\mu)\, Y_\nu(\mu)\;, \tag{6.1}$$

where Y_ν and M are μ-dependent. (We do not take into account the running of the Higgs vev.) m_ν is the mass matrix of the three light neutrinos as obtained from block-diagonalizing the complete 6×6 (for 3 singlet neutrinos) neutrino mass matrix, following the standard see-saw calculation. The energy-dependent mixing parameters are obtained from $m_\nu(\mu)$ and the running charged lepton

Yukawa matrix $Y_e(\mu)$. Between the see-saw scales or in a type II see-saw, there is an additional contribution to m_ν from the dimension 5 neutrino mass operator.

6.1.3 How can I obtain the CP asymmetry for leptogenesis?

The CP asymmetry in the case of thermal leptogenesis (in the limit $M_1 \ll M_2, M_3$) is implemented as output function in `REAP`. It can be obtained in the SM and MSSM by

```
RGEGetSolution[M_1, RGE\[Epsilon]1,1] .
```

The CP asymmetry is not implemented for other leptogenesis scenarios. However, the relevant quantities can be obtained via `RGEGetSolution`. See the notebook `RGETemplate.nb` for an example.

6.2 Implementation Details

6.2.1 I added the "SM" with `RGEAdd[‘‘SM’’,RGECutoff->1000]`, but `RGESetOptions[‘‘SM’’,RGEλ->0.3]` does not have any effect. Is this an error?

This is no error, but sometimes the EFT's are changed in such a way that the whole model is consistent. In this case the "SM" was changed to "SM0N", because all right-handed neutrinos are integrated out above 1000 GeV. You can use `RGESetOptions[‘‘SM*’’,RGEλ->0.3]` to change `RGEλ` in "SM" and "SM0N" at the same time. Then you do not have to care whether all neutrinos have been integrated out.

6.2.2 I want to change the Standard Model Higgs vev in all EFT's.

You can use wildcards with `RGESetOptions`, `RGESetEFTOptions`, `RGESetModelOptions`, `RGEGetOptions`, `RGEGetEFTOptions` and `RGEGetModelOptions`, because the name you enter is matched with `StringMatchQ`. See the documentation of Mathematica for the possible wildcards.

6.2.3 RGEAll does not work in "*0N"

`RGEGetSolution[Scale,RGEAll]` returns all parameters used in a see-saw model. Thus, Y_ν and M_{ν_R} are returned in addition to the parameters valid in a model without right-handed neutrinos. However, an error message will be produced, unless there are right-handed neutrinos at a higher scale, because `RGEGetSolution` obtains the values of Y_ν and M_{ν_R} recursively by determining the values at the cutoff. RGERaw which returns the parameters valid in "*0N" can be used instead of RGEAll, if Y_ν and M_{ν_R} are not defined.

6.2.4 Sometimes there are errors when RGESolve is executed twice.

`RGESolve` adds new EFT's to the model. This is in conflict with the automatic detection of transitions.
The simplest workaround is either to set `RGERemoveAutoGeneratedEntries`, an option of `RGESolve`, to 'True' (This is the default value of this option.) or if this does not help, make sure that `RGEReset` is executed before `RGESolve` is executed again.

6.2.5 `RGEGetSolution` does not return the leptogenesis parameters at the lightest right-handed neutrino mass.

In order to get e.g. the CP asymmetry ϵ_1 at the mass of the lightest right-handed neutrino MR1, use

```
RGEGetSolution[MR1,RGE\[Epsilon]1,1]
```

The additional `,1` tells `RGEGetSolution` to use the EFT valid immediately above the energy MR1 for returning the value of RGEϵ1. This is necessary because the leptogenesis parameters are not defined in the EFT without right-handed neutrinos that is valid below MR1 and that would be used by `RGEGetSolution` by default.

6.2.6 What will happen, if RGEye, RGEyμ and RGEyτ are passed to RGESetInitial in addition to the matrix RGEYe?

Generally, the matrices will be taken first and only if there is no matrix specified, the matrix is built from the specified eigenvalues and angles. In particular, `RGEYe` defines the Yukawa coupling matrix of the charged leptons if specified, and the eigenvalues `RGEye` etc. are not used then. The same applies for all other Yukawa coupling matrices.

6.2.7 Changing the value of RGEθ12 doesn't have any effect.

If the parameter RGEMνr is specified in `RGESetInitial`, it determines the effective mass matrix m_ν of the light neutrinos (together with RGEYν) via the see-saw formula. Therefore, all options affecting m_ν such as RGEθ12, `RGEMlightest` etc. do not have any effect in this case. If you would like to use these options, you have to remove the replacement rule for RGEMνr.

References

[1] T. Blazek, S. Raby, and S. Pokorski, Phys.Rev. **D52**, 4151 (1995), hep-ph/9504364.